Lec 21: 04/08/2010

Recombination (Closer Look):

Let us have a closer look at the recombination epoch and different processes that are relevant.

First consider the following process:

$e + p \longleftrightarrow H + \gamma$

One has to find the rate at which an electron with average kinetic energy $\langle \frac{1}{2} m_e v^2 \rangle = \frac{3}{2} kT$ is bound to a proton to form a Hydrogen atom. The thermally averaged cross section is given by:

$\langle \sigma_{rec} |\vec{v}| \rangle$

Where $\vec{v}$ is the relative velocity between e and p, which is practically the velocity of the electrons (protons are essentially at rest because of much higher mass).

This can be expanded as follows:

$$\langle \sigma_{rec} |\vec{v}| \rangle = \sum_{n=1}^{\infty} \langle \sigma_n |\vec{v}| \rangle$$

The n-th term is the sum represents the cross section for the electron to end up in the n-th energy eigenstate of the Hydrogen atom (n=1 being the ground state).

Recall that $E_n = \frac{m_e e^4}{2\hbar^2 n^2}$ and $\langle r \rangle_n \sim n a_0 = \frac{\hbar^2 n}{m_e e^2}$.

It is found that:

$$\boxed{\langle \sigma_n |\vec{v}| \rangle = \frac{4\pi^2 \alpha}{m_e^2} \frac{B}{n(3 m_e T)^{1/2}}} \quad (*)$$

Where $\alpha = \frac{e^2}{4\pi\hbar c}$ is the fine structure constant ($\alpha \simeq \frac{1}{137}$) and $B = 13.6$ eV is the ground state binding energy.

To calculate $\langle \sigma_n |\vec{v}| \rangle$ one has to find the overlap between the wavefunction of a free electron and the wavefunction of the n-th energy eigenstate.

The free electron wave function is a plane wave $e^{\frac{i\vec{p}\cdot\vec{r}}{\hbar}}$, where $|\vec{p}| = m_e |\vec{v}|$. Note that the electron will get bound to the proton if it gets close enough to it to feel the potential energy. Thus, what actually matters is the inner product of a plane wave and the n-th eigenstate wave function within a distance $r_0 \sim \frac{e^2}{2\pi \varepsilon_0 m_e |\vec{v}|^2}$ which after thermal averaging is $r_0 \sim \frac{e^2}{\varepsilon_0 kT}$.

Based on this one can explain some qualitative features in (*), such as the general dependence on m_e, T, n.

It is clear that the contribution from n=1 term dominate and hence:

$$\boxed{\langle \sigma_{rec} |\vec{v}| \rangle \sim \frac{4\pi^2 \alpha}{m_e^2} \frac{B}{3(m_e T)^{1/2}} = 4.7 \times 10^{-24} \left(\frac{1 eV}{T}\right)^{1/2} cm^2 \quad \left[4.7 \left(\frac{1eV}{T}\right)^{1/2} barn\right]}$$

From this one can find Γ_{rec}. As long as $\Gamma_{rec} > H$, the fractional ionization follows the Saha ionization equation

$$\frac{X_e^{eq}}{1-(X_e^{eq})^2} = \frac{4\sqrt{2}}{\sqrt{\pi}} \zeta_{(3)} \eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)$$

For $X_e^{eq} \ll 1$, we have:

$$X_e^{eq} \simeq \left(\frac{\sqrt{\pi}}{4\sqrt{2}}\right)^{1/2} \left[\frac{1}{\zeta(3)} \eta^{-1}\right]^{1/2} \exp\left(\frac{-B}{2T}\right)$$

It turns out that $\Gamma_{rec} \lesssim H$ at a redshift:

$$\boxed{z \simeq 1080 - 1180 \quad (\text{freeze out of } e + p \leftrightarrow H + \gamma)}$$

This results in a residual ionization $X_\infty \sim 3 \times 10^{-3}$.

Now, let us consider the other important process:

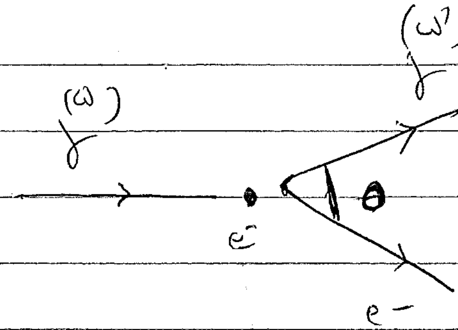
$e + \gamma \to e + \gamma$

The rate for this process is $\Gamma_\gamma = n_e \sigma$, where n_e is the number density of free electrons, and we have used the fact that the relative velocity between a photon and an electron is 1 (in natural units). Note that the electron has a very small average velocity at temperatures of O(eV).

The cross section for electron-photon scattering can be

Computed in QED:

$$\frac{\omega'}{\omega} = \frac{1}{1+\frac{\omega}{m_e}(1-\cos\theta)}$$



Here ω and ω' denote the frequency (which is the same as energy in natural units) of the incoming and outgoing photons respectively.

The partial cross section is given by the Klein-Nishina formula:

$$\boxed{\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right]}$$

In the limit that $\omega \ll m_e$, we find $\omega' \simeq \omega$, and:

$$\frac{d\sigma}{d\theta} \simeq \frac{\pi\alpha^2}{m_e^2}(1+\cos^2\theta) \Rightarrow \boxed{\sigma = \sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \quad \left(\tfrac{2}{3}\text{ barn}\right)}$$

As expected, at low energies the scattering is the same as Thomson scattering with a total cross section of $\frac{2}{3}$ barn.

Using this (and n_e from Saha equation) it turns out that $\Gamma_\gamma \lesssim H$ at:

$$\boxed{z_{dec} \simeq 1100 - 1200}$$

At this time photons can freely move without further scatterings off electrons. This happens slightly later than recombination when $T_{dec} \sim 0.26\ eV$ ($t_{dec} \sim 500{,}000$ yr).

Note that before recombination baryons are kept in equilibrium by scatterings off electrons. The baryon-photon scattering cross section is very low (it obeys the same expression as Klein-Nishina formula with m_e replaced by m_p, which is ~1000 times larger). Electrons themselves are in equilibrium with photons via Thomson (and at earlier time Compton) scattering.